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Technical Note

# On the fractal description of active nucleation site density for pool boiling

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## 1. Introduction

The research on the effects of the roughness of heating surfaces on pool boiling has made great advances in the last several decades. A number of quantitative relations for the description of nucleation site density have been developed. Mikic [1] and Lorez [2] proposed the following mathematical model:

$$N = c_l \left( \frac{r_l}{r_{\min}} \right)^m, \tag{1}$$

where $r_l$ is the mouth radius of the cavity when nucleation site density $N = 1$, $c_l$ a dimensional constant (1/unit area), $m$ a constant characterizing the boiling surface, and $r_{\min}$ is the minimum radius of the nucleation site defined as

$$r_{\min} = \frac{2\sigma T_{\text{sat}} v_{\text{fg}}}{h_{\text{fg}} \Delta T}. \tag{2}$$

This model did not include the dynamic relation between the cavity radius and the minimum radius of the nucleation site when boiling surface superheat varies. Based on the entrapping-vapor model proposed by Bankoff [3], Lorenz [4], Yang and Kim [5] established the dependence of the nucleation site density and the characteristic of a boiling surface with the aid of statistical analysis approach

$$N = \overline{N} \int_{r_{\min}}^{r_{\max}} \lambda e^{-\lambda r}\, dr \int_0^{\theta/2} \frac{1}{\sqrt{2\pi s}} \exp\left[ -\frac{(\beta - \overline{\beta})^2}{2s} \right] d\beta. \tag{3}$$

All the parameters in this model have obvious physical interpretation, and can be measured by suitable apparatus. The predictive results of this model agree well with the experimental in the smaller surface superheat range.

Fractal theory is a kind of nonlinear science. The object is to analyze a system, which is in the nature of the scaling invariance when the measure scale changes, to reveal the regularity, the gradation and the definition of a macroscopic system behind complex irregular appearance. The fractal features of a solid surface have been reported under various kinds of scales: from distributions of the diameters of the holes on planets and satellites [6], to the surface structure of protein [7]. Kang [8] proposed the methods to calculate the fractal dimension of the surface of inorganic material. Majumdar and Tein [9] describe the rough surface with the help of fractal geometry, and obtained the equation about the cavity distribution on the surface as

$$N(R > r) = \left( \frac{r_{\max}}{r} \right)^D \quad \text{in } r_{\min} < r < r_{\max}, \tag{4}$$

where $N$ is the number of the cavities on the rough surface. $r_{\max}$ and $r_{\min}$ are defined by Hsu's model [10]. This method with the single fractal dimension, $D$, could describe the geometrical character of a boiling surface of rigid self-similarity, but could not take account of the dynamic characteristics produced by the complicated non-self-similarity geometrical structure or the physical quantity distribution on this kind of structure. This is too simple to explain the complex boiling heat transfer phenomena, and hence, it is difficult to predict the properties of the phenomena. As compared with Bankoff–Lorenz's entrapping-vapor model, $r_{\max}$ and $r_{\min}$ in Eq. (4) consider only the cavity radius and overlooks concrete structure of the cavity. The obtained result will be the cavity distribution, not the active nucleation site distribution.

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**Nomenclature**

| | |
|---|---|
| $D$ | fractal dimension |
| $D_b$ | boiling dimension |
| $h$ | height (m) |
| $h_{fg}$ | latent heat (W $kg^{-1}$ $K^{-1}$) |
| $N$ | active nucleation site density (site/$mm^{-2}$) |
| $R$ | radius (m) |
| $r$ | radius (m) |
| $s$ | standard deviation |
| $T$ | temperature (K) |
| $\Delta T$ | boiling surface superheat (K) |
| $v$ | specific volume ($m^3$ $kg^{-1}$) |
| $\beta$ | half of cone angle |
| $\theta$ | contact angle |
| $\lambda$ | statistical parameter |
| $\rho$ | density (kg $m^{-3}$) |
| $\Delta\rho$ | density difference between liquid and saturated vapor (kg $m^{-3}$) |
| $\sigma$ | surface tension (N $m^{-1}$) |
| *Subscripts* | |
| c | at the mouth of the cavity |
| $l$ | radius defined in Eq. (1) |
| max | maximum value |
| min | minimum value |
| fg | liquid–vapor phase |

The present work analyzes the effects of heating surface feature on the active nucleation site density, so as to establish the relationship between them under the scale of fractal dimension.

## 2. Fractal dimension and boiling dimension

The fractal nature of a given surface of non-rigid self-similarity structure is limited only in a range of some scale, the geometric structure of a given physical problem needs to be described exactly as so-called 'valid dimension'. According to the definition, when the scale changes the dependence of fractal dimension, $D$, on the measuring scale $r$ and the number obtained on this scale $N(r)$ is

$$D(r) = -\frac{\ln N(r)}{\ln r}. \tag{5}$$

If $N(r)$ is not a special function, the right-hand side of Eq. (5) can not be a constant. It is difficult to use a single fractal dimension for a certain physical problem. Takayasu defined generalized fractal dimension as follows [11]:

$$D(r) = -\frac{\mathrm{d}\,\ln N(r)}{\mathrm{d}\,\ln r}. \tag{6}$$

Solving Eq. (6), yields

$$N(R) = N(r)\exp\left(-\int_r^R \frac{D(s)}{s}\,\mathrm{d}s\right). \tag{7}$$

This relates the fractal dimension to the numbers of cavities $N(r)$ and $N(R)$ on the measuring scales $R$ and $r$.

Following Bankoff–Lorenz's entrapping-vapor model as the condition of cavity entrapping vapor, the boiling dimension of boiling heat transfer surface, $D_b$, is defined as

$$D_b = -\frac{\ln N(r)}{\ln r}, \tag{8}$$

where $N(r)$ is the number of the cavities (Fig. 1) satisfying Bankoff–Lorenz's entrapping-vapor model, whose radius is greater than $r$, and whose cone angle $\beta \leqslant \theta/2$. For a fixed liquid–solid interface, the contact angle varies a little with the change of the boiling surface superheat. However, if the change of the contact angle is neglectable, the boiling dimension $D_b$ will be fixed under the condition of a given scale.

When a scale, $r_{min}$, is used to measure the nature of a surface, the cavity with the mouth radius less than $r$ cannot be measured, thus, to satisfy the condition that the nucleation site is active. The mouth radius of all the measurable cavities would be greater than $r_{min}$, and the upper limit of the measuring scale $r_{max}$ can be given as [5]:

$$r_{max} = \sqrt{4\sigma\cos^2(\theta-\beta)/((\Delta\rho)g[1-\sin(\theta-\beta)]\{1+(\sin^2(\theta-\beta)/3\cos(\theta-\beta))\})}. \tag{9}$$

Under the condition of $\beta \leqslant \theta/2$, the numbers of cavities $N(r_{max})$ and $N(r_{min})$ corresponds to the measuring scales

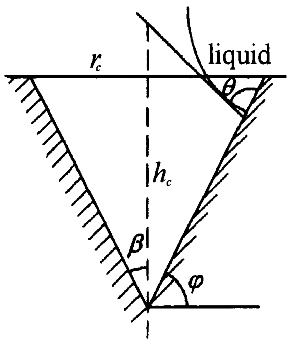


Fig. 1. Schematic diagram of a cavity.

$r_{max}$ and $r_{min}$, respectively. For a given boiling surface superheat, the number of the active nucleation site is $N(r)$ corresponding to the measuring scale $r_{min}$, and $N(r_{max}) \to 0$ because $r_{max}$ is far greater than the mouth radius of the cavities on the surface produced by the conventional machining process. Thus

$$N(r) = N(r_{min}) - N(r_{max}) \to N(r_{min}). \tag{10}$$

That is, the number of the active nucleation sites $N(r)$ satisfies the condition $\beta \leqslant \theta/2$ and measured with the scale $r_{min}$. From Eqs. (6) and (8), one has

$$N(r) \propto \bar{r}_{min}^{-D_b(r)}, \tag{11}$$

where $r_{min}$ is the relative measuring scale. Eq. (11) can be rewritten to dimensionless form as

$$N(r) = c\left(\frac{r_{min}}{r_0}\right)^{-D_b(r)}. \tag{12}$$

Here $r_0$ is a characteristic roughening length, a pure geometrical quantity, which reflects the precision of the fractal nature of an object, depending on the kind of solids. The constant $c$ is the number of nucleation sites under the scale $r_{min}$, that is $c = N(r_0)$, which can be obtained by measurement equation (12) has obvious physical interpretation, and is useful in more wide range.

Substituting Eq. (2) into Eq. (12), the dependence of the number of nucleation sites per unit area on the boiling surface superheat, i.e.,

$$N(r) = N(r_0)\left(\frac{2\sigma T_{sat} v_{fg}}{r_0 h_{fg} \Delta T}\right)^{-D_b(r)}. \tag{13}$$

For a given system of liquid–solid surface, $2\sigma T_{sat} v_{fg}/h_{fg} = k$ is a constant, so that

$$N(r) = N(r_0)(r_0)^{D_b(r)}(k/\Delta T)^{-D_b(r)}. \tag{14}$$

Then, boiling dimension, $D_b(r)$, can be measured by way of changing the scale. That is, various scales are used to measure the distributions of the number of active nucleation sites in a fixed surface, the statistical mean value can be calculated by means of random sampling.

## 3. Examination of the model

An experimental system of stainless steel and water is employed in the present investigation. The stainless steel F-5-11 is grounded with wet fine abrasive paper and polished. The numbers and mouth radius of the cavities are observed with a JSM-35CF type of scanning electron microscope. The depths of the cavities are measured with a differential interference contrast microscope of Nomarski type. The cavities are approximately considered to be conical. The cone angle in Bankoff–Lorenz's entrapping-vapor model can be calculated by using trigonometric function $\beta = \text{arctg}(r_c/h)$. When water is heated, the contact angle of liquid and the surface is $\theta = 65°$. Micrometer is used as the measuring scale to make the measured data dimensionless. By Eq. (5), the statistical average values are used to calculate the boiling fractal dimension.

The result of the calculation on logarithmic coordinates is shown in Fig. 2, from which the number of cavities satisfying Bankoff–Lorenz's entrapping-vapor model appear to be a good linear relation with the measuring scale, if $r_{min}$ is fairly large enough. This shows that boiling dimension will be a fixed value correspondingly.

The experiments on the boiling of water are carried out at the atmospheric pressure. Substituting the boiling dimension obtained from Fig. 2 into Eq. (12), the active nucleation sites can be calculated and the result is shown in Fig. 3. It is seen from Fig. 3 that the predictive values agree well with the experimental data when the boiling surface superheat is less than 2.6°C. When the boiling surface superheat is greater than 2.6°C, it is difficult to determine the number of bubbles visually on the picture, but as shown in Fig. 3, the increment of the number of the active nucleation sites seems slowly increased with increasing boiling surface superheat, and the active nucleation site density surely cannot be infinite.

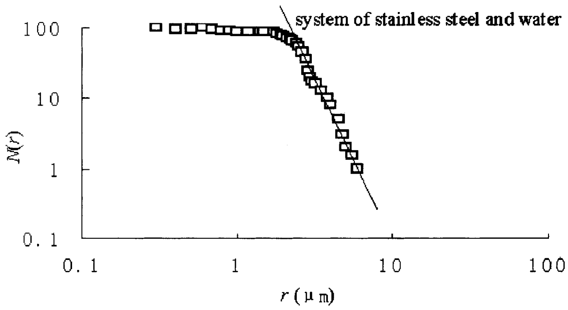


Fig. 2. Distribution of cavity diameters.

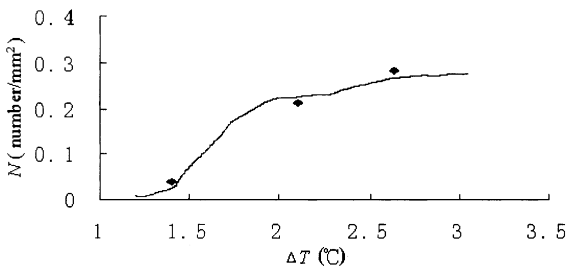


Fig. 3. Dependence of $\Delta T$ on $N$.

## 4. Conclusions

1. The active nucleation site density of a heating surface of pool boiling has a feature of fractal.
2. The relationship between the active nucleation site density of pool boiling and its boiling dimension is expected to be governed by Eq. (14). When boiling surface superheat is less than 2.6, the predicted results are agreed well with the results of the experiment.

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